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NUMERICAL MODELING OF UNSTEADY RADIATIVE-CONVECTIVE HEAT TRANSFER ON A FLAT-PLATE BOUNDARY LAYER IN A SELECTIVELY EMITTING AND SCATTERING MEDIUM

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A conjugation problem for radiative-convective heat transfer in a turbulent flow of a high-temperature gas–particle medium around a thermally thin plate is considered. The plate experiences intense heating from an outside source that emits radiation in a restricted spectral range. Unsteady temperature fields and heat-flux distributions along the plate are calculated. The results permit prediction of the effect of the type and concentration of particles on the dynamics of the thermal state of both the medium in the boundary layer and the plate itself under conditions of its outside heating by a high-temperature source of radiation.

In the present paper, an attempt is made to develop an adequate model for radiative-convective heat transfer in a turbulent boundary layer of a high-temperature gas–particle medium over a solid surface. The problem of interest may be used in heat-transfer calculations of steam-boiler furnaces, channels of MHD-generators, various chemical apparatus, thermal-protection systems of space vehicles, etc.

The present consideration is based on the approach previously used in [1–3], where the radiative-convective heat transfer on a thermally thin plate was studied in a conjugate statement of the problem. Further, we take into account the presence of particles in the flow, which necessitates an adequate consideration of radiation scattering by solid particles in calculating the radiative heat transfer.

We consider a conjugation problem for radiative-convective heat transfer in a turbulent flow of a medium that emits, absorbs, and scatters radiation over a thermally thin plate. It is assumed that the particles in the flow exert no influence on the thermal properties of the medium but affect its optical properties, which, in addition, depend on temperature and on the wavelength of radiation. The heat capacity is assumed to be temperature-independent, whereas the viscosity and thermal conductivity are linear functions of temperature, and the density is inversely proportional to it. The radiation transfer along the plate is ignored. The time required for boundary-layer heating is assumed to be far shorter than that for the plate; hence, the quasi-stationary approximation for the boundary-layer heat transfer may be used. The plate is heated from an initial temperature T_{w0}, the temperature over the length $0 < x < x_0$ of the plate being kept unchanged during the whole heating period. The lower surface and the trailing edge of the plate are thermally insulated. Outside the boundary layer, there is a source of radiation in the form of a black surface with a temperature T_s. The surface emits radiation in a restricted spectral range Δ, in which the medium is nontransparent. The radiating surface of the source is parallel to the plate.

The thermal state of the plate is described by the nonstationary heat-conduction equation, and the boundary-layer heat transfer is described by the well-known system of equations that includes the equations of continuity, motion and energy.

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With the Dorodnitsyn transform, the dynamic part of the problem can be solved independently of the thermal one, and with allowance for the adopted assumption, it reduces to solving the differential equation

$$((1+\mu_{\rm t})f'')' + \frac{1}{2}\,ff'' = \xi\Big(f'\,\frac{\partial f'}{\partial\xi} - f''\,\frac{\partial f}{\partial\xi}\Big) \tag{1}$$

with the following boundary conditions: $f = 0$ and $f' = 0$ for $\eta = 0$ and $f' \to 1$ as $\eta \to \infty$. Here f is the dimensionless stream function, $\eta = \Big(\dfrac{\rho_\infty u_\infty}{\mu_\infty x}\Big)^{1/2}\displaystyle\int_0^y \frac{\rho}{\rho_\infty}\,dy$ and $\xi = x/L$ are the dimensionless transverse and longitudinal coordinates, respectively, x and y are the corresponding dimensional coordinates, u is the longitudinal velocity, ρ is the density, μ is the viscosity, and L is the calculation domain (length) of the plate. The prime denotes differentiation with respect to the coordinate η; the subscript ∞ refers to the free-stream conditions.

The thermal part of the problem is represented by the equations and boundary conditions that describe heat transfer in the boundary layer:

$$\begin{gathered}\frac{\partial}{\partial\eta}\Big(\Big(\frac{1}{\rm Pr} + \frac{\bar\mu_{\rm t}}{\rm Pr_t}\Big)\frac{\partial\theta}{\partial\eta}\Big) + \frac{f}{2}\,\frac{\partial\theta}{\partial\eta} - \xi f'\,\frac{\partial\theta}{\partial\xi} - \frac{\rm Sk}{\rm Re\,Pr}\,\xi\Psi = 0, \quad \xi_0 < \xi < \xi_1, \quad 0 < \eta < \infty,\\ \xi = \xi_0\colon\ \ \theta = \theta_0, \qquad \eta = 0\colon\ \ \theta = \theta_w, \qquad \eta \to \infty\colon\ \ \theta \to 1\end{gathered} \tag{2}$$

and in the plate:

$$\begin{gathered}\frac{\partial\theta_w}{\partial{\rm Fo}} = \frac{\partial^2\theta_w}{\partial\xi^2} - æ\,{\rm Sk}\,Q_w, \qquad \xi_0 < \xi < \xi_1, \qquad {\rm Fo} > 0,\\ {\rm Fo} = 0\colon\ \ \theta_w = \theta_{w0}, \qquad \xi = \xi_0\colon\ \ \theta_w = \theta_{w0}, \quad \xi = \xi_1\colon\ \ \frac{\partial\theta_w}{\partial\xi} = 0.\end{gathered} \tag{3}$$

Hereinafter $\bar\mu_{\rm t} = \mu_{\rm t}/\mu$, $\mu_{\rm t}$ is the turbulent viscosity, $\theta = T/T_\infty$ is the dimensionless temperature, $\theta_0(\eta)$ is the self-similar solution of the energy equation (2) without emission, $æ = \lambda_\infty L/(\lambda_c H)$ is the conjugation parameter (H is the plate thickness), ${\rm Re} = \rho_\infty u_\infty L/\mu_\infty$, ${\rm Fo} = a_c t/L^2$, ${\rm Pr} = \mu_\infty/(\rho_\infty a_\infty)$, and ${\rm Sk} = 4\sigma T_\infty^3 L/\lambda_\infty$ are the Reynolds, Fourier, Prandtl, and Stark numbers, respectively, ${\rm Pr_t}$ is the turbulent Prandtl number, λ_c and λ_∞ are the thermal conductivities of the plate and medium in the free flow, respectively, a_c is the thermal diffusivity in the plate material, T_∞ is the free-stream temperature, $\xi_0 = x_0/L$, $\xi_1 = x_1/L$, x_0 and x_1 are the end points of the calculation domain, and σ is the Stefan–Boltzmann constant; the subscript w refers to the conditions on the plate.

The dimensionless total wall heat-flux density Q_w on the plate in Eq. (3) is given by the expression

$$Q_w = -\frac{1}{\rm Sk}\Big(\frac{\rm Re}{\xi}\Big)^{1/2}\frac{\partial\theta}{\partial\eta}\Big|_{\eta=0} + \Phi_w,$$

where $\Phi_w = E_w/(4\sigma T_\infty^4)$ and E_w is the total density of the resultant radiation flux on the plate.

The expression for the dimensionless divergence of the radiation–flux density in Eq. (2) is

$$\Psi = \int_\Delta \frac{\tau_{\lambda L}(E_{0\lambda} - E_{*\lambda})}{4\sigma T_\infty^4}\,d\lambda, \tag{4}$$

where $E_{0\lambda}(T) = 2\pi hc^2/[\lambda^5(\exp{(hc/(k\lambda T))} - 1)]$ is the equilibrium radiation–flux density, $E_{*\lambda} = 2\pi\displaystyle\int_{-1}^{1} I_\lambda(\tau_\lambda,\gamma)\gamma\,d\gamma$ is the volume density of incident radiation, I_λ is the radiation intensity, γ is the cosine of the angle between the ordinate axis and the direction of propagation of radiation, λ is the wavelength, c is the velocity of light in vacuum, h and k are the Planck and Boltzmann constants, respectively, $\tau_{\lambda L} = k_\lambda L$ is the characteristic optical thickness, and k_λ is the damping factor of the medium (the subscript λ marks spectral quantities). Integration over the wavelength in Eq. (4) is performed in the spectral range Δ, in which the medium is nontransparent. The optical depth in the cross section ξ of the boundary layer can be written as

$$\tau_\lambda = \left(\frac{\xi}{\mathrm{Re}}\right)^{1/2} \int\limits_0^\eta \frac{\tau_{\lambda L}}{\theta}\, d\eta$$

and depends on the wavelength and temperature.

Radiative heat transfer in the system under consideration, i.e., in a plane slab, bounded by the source and plate surfaces, of a medium that emits, absorbs, and scatters radiation, is described by the radiative heat-transfer equation. To solve this equation, we use the method of average fluxes [4]. In this method, the calculation of the radiation field reduces to solving the system of equations

$$\frac{d}{d\tau_\lambda}\,(\Phi_\lambda^+ - \Phi_\lambda^-) + (1-\omega_\lambda)(m_\lambda^+\Phi_\lambda^+ - m_\lambda^-\Phi_\lambda^-) = (1-\omega_\lambda)\Phi_{0\lambda}(T),$$

$$\frac{d}{d\tau_\lambda}\,(m_\lambda^+\delta_\lambda^+\Phi_\lambda^+ - m_\lambda^-\delta_\lambda^-\Phi_\lambda^-) + (1-\omega_\lambda\bar{\zeta})(\Phi_\lambda^+ - \Phi_\lambda^-) = 0$$

with the boundary conditions

$$\tau_\lambda = 0\colon\quad \Phi_\lambda^+(0) = \varepsilon_w\Phi_{0\lambda}(T_w) + (1-\varepsilon_w)\Phi_\lambda^-(0), \qquad \tau_\lambda = \tau_{\lambda\infty}\colon\quad \Phi_\lambda^-(\tau_{\lambda\infty}) = \Phi_{0\lambda}(T_s).$$

Here $\Phi_\lambda^\pm = 2\pi \int\limits_{0(-1)}^{1(0)} \frac{I_\lambda(\tau_\lambda,\gamma)}{4\sigma T_\infty^4}\,\gamma\, d\gamma$ is the dimensionless density of a hemispherical flux of radiation, $\Phi_{0\lambda}(T) = E_{0\lambda}(T)/(4\sigma T_\infty^4)$ is the dimensionless density of equilibrium radiation, $m_\lambda^\pm = \int\limits_{0(-1)}^{1(0)} I_\lambda(\tau_\lambda,\gamma)\, d\gamma \Big/ \int\limits_{0(-1)}^{1(0)} I_\lambda(\tau_\lambda,\gamma)\gamma\, d\gamma$ and $\delta_\lambda^\pm = \int\limits_{0(-1)}^{1(0)} I_\lambda(\tau_\lambda,\gamma)\gamma^2\, d\gamma \Big/ \int\limits_{0(-1)}^{1(0)} I_\lambda(\tau_\lambda,\gamma)\, d\gamma$ are the heat-transfer coefficients [4], T_s is the temperature of the source, $\tau_{\lambda\infty} = \left(\frac{\xi}{\mathrm{Re}}\right)^{1/2} \int\limits_0^{\eta_\infty} \frac{\tau_{\lambda L}}{\theta}\, d\eta$ is the optical thickness of the boundary layer, ε_w is the emissivity of the plate, $\omega_\lambda = \beta_\lambda/k_\lambda$ is the single-scattering albedo, k_λ and β_λ are the damping and scattering factors, respectively, and $\bar{\zeta}$ is the shape factor for the phase function [5].

To calculate the velocity field in the turbulent boundary layer, we used the Cebeci–Smith two-layer model [6].

Equation (1) was integrated by the iteration-difference method. The thermal part of the problem was solved by consecutive adjustment of the plate temperature on the basis of simultaneous solution of the energy and radiation-transfer equations with the boundary condition at the interface, depending on the sought temperature. This procedure was described in detail in [7].

We studied a gas–particle medium, which was a mixture of carbon dioxide, steam, and solid particles. As the solid component, coal and ash particles were considered. To some extent, the atmosphere inside steam-boiler furnaces can be modelled with such a mixture.

Ignoring scattering in the gaseous phase, we can represent the damping factor of the model medium as $k_\lambda = k_{\lambda p} + æ_{\lambda g}$, where $k_{\lambda p}$ is the damping factor of the cloud of particles and $æ_{\lambda g}$ is the gas-absorption coefficient.

Selective absorption of radiation in the gaseous phase was taken into account by the narrow-band method based on the Goody statistical model [8]. In this model, the distribution of absorption lines over the frequency spectrum is assumed to be random, and the intensity of each line obeys a certain law (in most cases, an exponential one). In this method, the spectral absorption coefficient at moderate pressures can be represented as

$$æ_{\lambda g} = P(\gamma_{\lambda \mathrm{CO_2}} C_{\mathrm{CO_2}} + \gamma_{\lambda \mathrm{H_2O}} C_{\mathrm{H_2O}}),$$

where P is the total pressure of the gas, C are the molar concentrations of the components of the mixture, and γ_λ is the average intensity of a line in the absorption band.

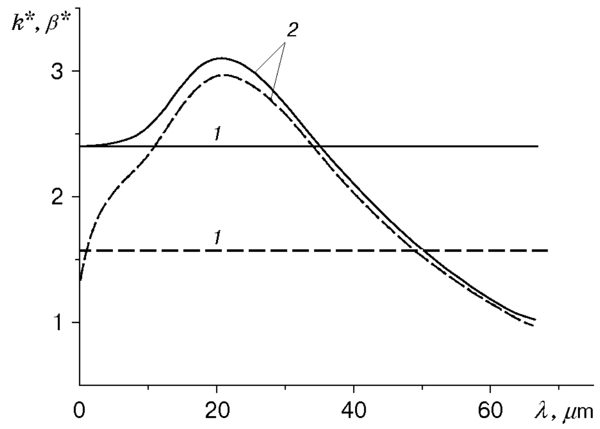

Fig. 1

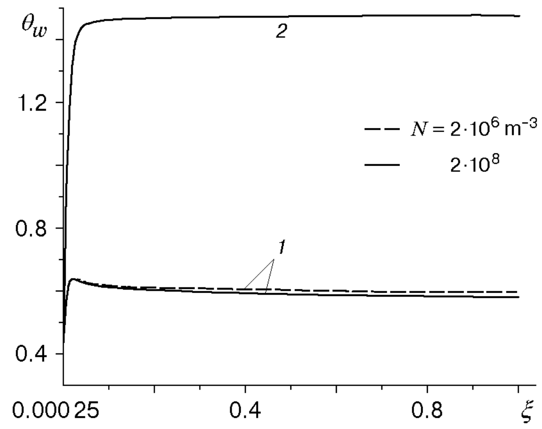

Fig. 2

The band parameter γ_λ is temperature-dependent. The values of this parameter within the temperature range of 300–1500 K were borrowed from [9, 10]. In calculating radiative heat transfer, we took into account the rotational band and the bands with absorption coefficients 7250, 5331, and 3755 cm^{-1} for H_2O, and 667 and 3715 cm^{-1} for CO_2.

The parameters that characterize the optical properties of the particles were borrowed from [11]. Considering the cloud of particles as a polydispersive mixture with a gamma-distribution over the size, Kim and Lior [12] obtained approximate formulas for estimating the damping and scattering factors as functions of the diffraction parameter $x = \pi\bar{d}/\lambda$ ($\bar{d}$ is the mean particle diameter). The expressions for the damping and scattering factors ($k_{\lambda p}$ and $\beta_{\lambda p}$, respectively) of coal particles are

$$k_{\lambda p} = 2\pi N\Big(\frac{\bar{d}}{2}\Big)^2 \frac{\alpha+2}{\alpha+1}, \qquad \beta_{\lambda p} = \pi N\Big(\frac{\bar{d}}{2}\Big)^2 \frac{\alpha+1}{\alpha+2}\Big(2 - \frac{f_1+f_2}{2}\Big). \tag{5}$$

Here N is the numerical concentration of particles, α is an empirical factor that characterizes the size distribution of particles, $f_i = 8[q_i - \ln(1 + q_i + q_i^2/2)]/q_i^2$ ($i = 1$ and 2), $q_1 = (nn')^{-1/2}$, and $q_2 = 2/q_1$ (n and n' are, respectively, the real and imaginary parts of the complex refractive index $m = n - in'$).

For ash particles, the damping factor can be calculated by the formula

$$k_{\lambda p} = 2\pi N b^{\alpha+1}\Big\{\frac{(\alpha+2)(\alpha+1)}{b^{\alpha+3}} - \frac{2(\alpha+1)\sin[(\alpha+2)\chi](\alpha+1)}{C_1(b^2+C_1^2)^{(\alpha+2)/2}} + \frac{2}{C_1^2 b^{\alpha+1}} - \frac{2\cos[(\alpha+1)\chi]}{C_1^2(b^2+C_1^2)^{(\alpha+1)/2}}\Big\}, \tag{6}$$

$$C_1 = 4\pi(n-1)/\lambda, \qquad b = (\alpha+1)/(\bar{d}/2), \qquad \chi = \arctan(C_1/b).$$

The absorption coefficient of ash is known to be small; hence, we can assume that the scattering factor due to ash is $\beta_{\lambda p} \approx k_{\lambda p}$. Formulas (5) and (6) describe the optical properties of the particles within the range $x = 25$–100 with an accuracy of 10%.

The heat-transfer calculations were carried out for the free-stream temperature $T_\infty = 1000$ K and the temperature of the outside source of radiation $T_s = 1500$ K. The following values of the determining parameters were assumed: $\theta_{w0} = 0.3$, $\mathrm{Pr} = 0.7$, $\mathrm{Pr_t} = 0.9$, $\mathrm{Re} = 10^6$, $\mathrm{Sk} = 5\cdot 10^5$, and $æ = 1$. The emissivity of the plate was $\varepsilon_w = 0.9$, which corresponds to an almost black surface. The concentration of particles N was varied within the range $2\cdot 10^6$–$2\cdot 10^8$ m^{-3}. The step in the dimensionless time (Fourier number) was $\Delta\mathrm{Fo} = 5\cdot 10^{-6}$.

Figure 1 shows the damping and scattering factors $k^* = k_{\lambda p}/(N\pi\bar{d}^2/4)$ and $\beta^* = \beta_{\lambda p}/(N\pi\bar{d}^2/4)$ (solid and dashed curves, respectively) calculated by formulas (5) and (6) for coal and ash particles (curves 1 and 2, respectively). We have $\bar{d} = 100$ μm, $\alpha = 4$, and $m = 2.02 - 0.8i$ for coal particles and $\bar{d} = 20$ μm, $\alpha = 4$, and $m = 1.5 - 0.01i$ for ash particles. The range of the wavelengths λ was 0.7–67 μm.

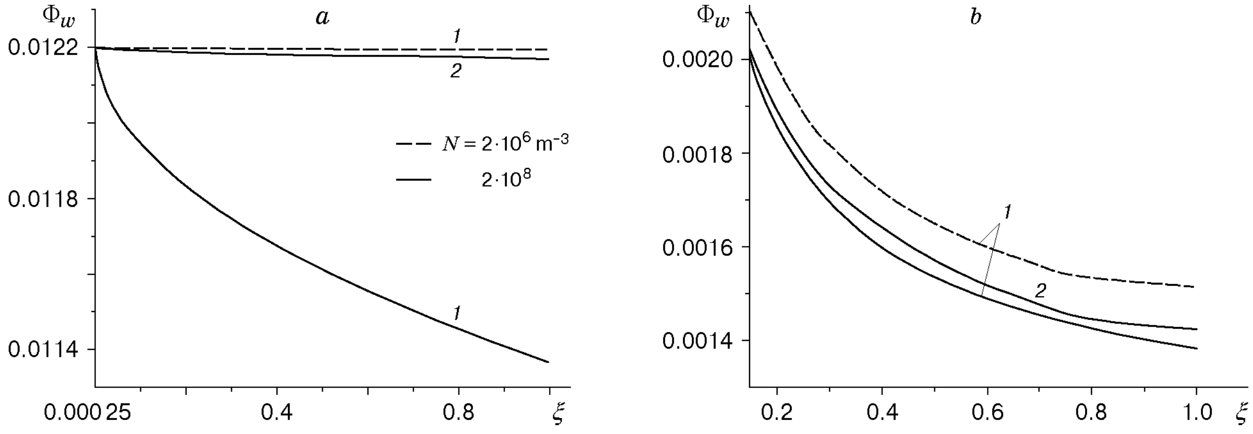

Fig. 3

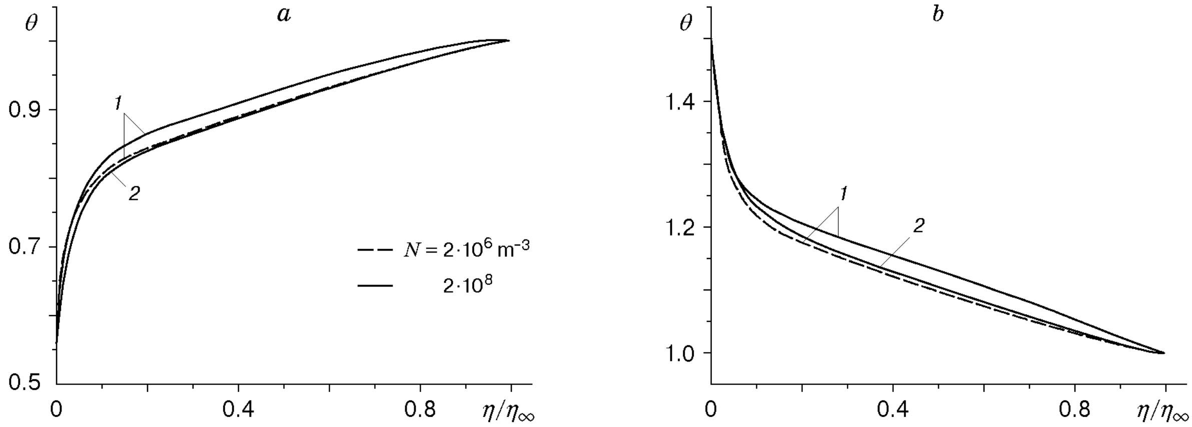

Fig. 4

Figure 2 shows the distribution of the temperature θ_w of an almost black plate ($\varepsilon_w = 0.9$) for two concentrations N of coal particles. Curves 1 were calculated with ten time steps, and curves 2 show the calculation results for the stationary regime. In the stationary regime, the curves for two values of N coincide. The particle concentration is seen to play a noticeable part only at the initial stage of heating. The latter can be explained by the fact that, under the stationary conditions, when the plate temperature is high, thermal emission from the plate contributes predominantly to the radiation flux. Emission from particles is much lower, since the mean temperature of the medium in the boundary layer is lower than the plate temperature. The temperature distribution for ash particles is similar.

Figure 3 shows the density distribution of the resultant radiation flux Φ_w along the surface of an almost black plate for two concentrations N of coal and ash particles (a and b are the calculation results for ten time steps and for the stationary regime, respectively). Curves 1 and 2 shows the calculated distributions for coal and ash particles, respectively. Curves 2 coincide for different values of N. A stronger dependence of Φ_w on the concentration of coal particles is noteworthy, which results from the fact that coal particles, which have a greater mean diameter $\bar{d}$ compared to ash particles, have a higher damping factor $\beta_{\lambda p} = \beta^*(N\pi\bar{d}^2/4)$ for an identical value of N.

Figure 4 shows the temperature profile at the last cross section ($\xi = 1$) of the boundary layer in the case of an almost black plate. The notation in Fig. 4 is the same as in Fig. 3.

Close consideration of the evolution of the distribution of the total heat flux Q_w along the surface of an almost black plate shows that the type of particles is of minor importance under stationary conditions, when thermal emission contributes predominantly to radiative transfer. The total heat flux for coal particles diminishes owing to a considerable decrease in emission caused by the larger diameter of these particles compared to ash particles.

The results obtained allow one to predict the effect of both the type of particles and their concentration on the dynamics of the thermal state of the medium in the boundary layer and of the plate itself under conditions of its heating by an outside high-temperature source of radiation.

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